Session B-4
Preprint 4301

Improved Triode Model

*Jonathan Scott, ***Stefan Roxenborg, and **Anthony Parker
*Dept. of Electrical Engineering, University of Sydney, NSW; ** Dept. of Electronics, Macquarie University, NSW
***Royal Institute of Technology, Stockholm, Sweden

Presented at

6th Australian Regional Convention

10th - 12th September 1996
World Congress Centre, Melbourne

AES

AN AUDIO ENGINEERING SOCIETY PREPRINT

Improved Triode Model

Jonathan Scott‡, AES Member, Stefan Roxenborg⋆, and Anthony Parker†

‡Department of Electrical Engineering, University of Sydney, Australia 2006
⋆Royal Institute of Technology, Stockholm, Sweden,
†Department of Electronics, Macquarie University, Australia 2109

Recent use of thermionic valves has grown to such a level that factories are being reopened, and many manufacturers offer equipment boasting the use of valves, commanding a great price. Interest in the design of valve equipment has lead to the appearance of CAD models of valves. We show that the theoretical model of a triode is a poor representation of its actual operation. We present an improved model. We show that our new model, based on experience gained modelling GaAs FETs, offers excellent agreement with values measured using two example tubes.

0 Introduction

Western Electric (AT&T) is reopening its vacuum tube plant, Westrex Tubes, in Kansas City, that closed in 1988.[1] Westrex state that "most high-end manufacturers offer a valve product". Recent interest in thermionic valves has grown to a level where factories are being reopened, while manufacturers offer equipment boasting the use of valves, and commanding a great price. Interest in the design of valve equipment has lead to the appearance of CAD models of valves.[2] One company offering a SPICE product reports a significant number of enquiries for valve models.

In this paper, we show that the generally-accepted theoretical model of a triode (the "standard" or conventional model), and thus the CAD model normally used with programs such as SPICE,[3, 4] is a poor representation of its actual operation. Next, we present a new model, based on experience gained modelling GaAs FETs. We will show that it offers excellent agreement with measured performance.

1 Theoretical Model of a Triode

The plate current of a triode is given in textbooks as

$$I_p = K(\mu_g V_g + V_p)^{\frac{3}{2}} \tag{1}$$

when $\mu_g V_g + V_p > 0$ and 0 otherwise, where V_g is the grid-cathode potential, V_p is the plate-cathode potential, and K and μ_g are model parameters associated with the individual valve. Equation 1 is usually presented as shown in figure 1.

A SPICE triode model thus consists of a voltage-controlled current source (Gxx card) implementing the above equation, plus fixed inter-electrode capacitances. (The conditional is sometimes implemented with diodes and/or BJTs.)

6BM8	K	μ_g	P	K_v	relative error
Standard	$\approx 3\mu$	54	1.5	-	0.000423
Enhanced	$\approx 28\mu$	64	1.07	-	0.000216
New	$\approx 26\mu$	66	1.09	22	0.000124

Table 1: Parameter values for the 6BM8 used in calculations shown in figures.

12AX7	K	μ_g	P	K_v	relative error
Standard	$\approx 1.7\mu$	79	1.5	-	0.0046
Enhanced	$\approx 9.3\mu$	88	1.18	-	0.0022
New	$\approx 8.0\mu$	91	1.21	28	0.0009

Table 2: Parameter values for the 12AX7 used in calculations shown in figures.

2 Comparison with Measured Values

Equation 1 may be fitted to measured values by straightforward numerical means. Using the Solver function in Excel, values for K and μ_g have been found for two sets of data measured from two representative triode valves, a 12AX7 and the triode in a 6BM8 triode-pentode. Figures 3 and 2 plot measured data against values calculated using the standard model, for these two valves. The fit is obviously rather crude. Values of the parameters K and μ_g for these valves may be found in tables 1 and 2. The tables give also parameter values relevant in the following sections, and relative error estimates that provide some sort of objective if simple measure of the closeness of the fit.

3 Enhanced Standard Model

Although classical physics predicts that the power relating voltage to plate current will be 1.5, it may be that this is not strictly true. For example, in solid-state semiconductor physics, the "ideality constant", n, is apt to vary with a number of factors, and is infrequently the expected value of 2. Can the standard model be used, simply by allowing another degree of freedom in the fit (allowing the power, P to vary)?

Figures 5 and 4 plot measured data against values calculated using the standard model, for these two valves, but with the power, P, treated as a variable instead of a constant. Note that the curvature in the main forward operating region is better represented. Although the fit is better, it remains weak especially in the region where the device approaches cutoff. The region where plate voltage is above 200V and the gate is at its most negative is conspicuously erroneous. (This is the equivalent of the so-called subthreshold region in a FET, where drain voltage is high but the gate is reverse biased close to cutoff.)

4 A New Model

The conventional triode model, derived from simple physics, is ineffective at representing the true operation of a real-world device. Similar circumstances are not uncommon in the world of modelling GaAs FETs, in particular MESFETs and HEMTs.[5, 6, 7] Drawing on the idea of "nested transforms", it is possible to improve the "fittability" of a model, and at the same time to increase simulation speed.[1] A suitable modification, borrowed from III-V semiconductor modelling, of equation 1 is

$$I_p = K_p\{K_v \ln[e^{(\frac{\mu_g V_g + V_p}{K_v})} + 1]\}^P \tag{2}$$

where again V_g is the grid-cathode potential, V_p is the plate-cathode potential, but K_p, K_v, μ_g, and now also P, are the model parameters associated with the individual valve. The elimination of the conditional associated with equation 1 also improves simulation speed. (Equation 2 returns $I_p > 0$ for all values of V_g and V_p.)

[1]The numerical methods embodied in the matrix engine of SPICE are seriously impeded by non-differentiability of functions, as is the case when conditionals occur.[8] This is overcome by the use of nested transforms.[7]

5 Performance of New Model

Figures 7 and 6 plot measured data against values calculated using the new model, for the same two valves. Parameter values and residual errors are again given in tables 1 and 2. The fit is greatly improved. In particular, note the close fit in the region approaching cutoff.

5.1 Simulation Example

Consider the circuit shown in figure 8. Using the conventional and new models yields very different predictions of THD at 1kHz. The predicted output spectra are shown in figure 9. THD of -72dB and -83dB respectively is predicted, while a value of less than -80dB is found by actual measurement.

6 Implications

We have observed that the $\frac{3}{2}$ power-law of the basic equation is far from realistic. Two things follow. One implication is that algebraic calculations, using equation 1, designed to minimise distortion by playing on the power law expected from theory, will be all but useless. The second implication is that the simple physics used to derive equation 1 misses out on much that is going on inside the valve.

7 Conclusion

The decline in the use of thermionic valves prior to the rise of powerful numerical tools for electronic circuit simulation left the tuning of characteristic equations to practical performance undone. We have shown that the effort devoted to this problem with modern, solid-state, field-effect devices can be applied to the triode with great success. We have presented a model suitable for SPICE implementation, and that describes actual triode valves with good accuracy. There are good prospects for applying numerical tools to the simulation, design and understanding of valve circuits.

A PSPICE Subcircuit

A subcircuit implementation for the 12AX7 suitable for use in PSPICE is given below.

```
* --------------------------
* triode subcircuit (12AX7)
.param KP=8u
.param KV=28
.param P=1.21
.param UG=91
.subckt V12ax7 a g k
Gplate a k
+ VALUE={KP*(KV*PWR(EXP(log((V(g,k)*UG+V(a,k))/KV)+1),P))}
Cgp a g 1.5p
Cgk k g 1.5p
Cak k a 0.5p
.ends
* --------------------------
```

References

[1] Barry Fox, "High fidelity ain't what it used to be", *New Scientist*, Technology, 3 February 1996, p18.

[2] W. Marshall Leach, Jr, "SPICE Models for Vacuum-Tube Amplifiers", *Journal of the Audio Engineering Society*, vol. 43, no. 3, March 1995, pp117–126.

[3] Charles E. Hymowitz and Lawrence G. Meares, *SPICE Application Handbook*, 2^{nd} Edition, IntuSoft, San Pedro, CA, 1994.

[4] A. Vladimirescu, K. Zhang, A.R. Newton and D.O. Pederson, *SPICE 2G User's guide*, University of California Berkeley, 1981.

[5] Jonathan Scott and Anthony Parker, "Trends in Device Characterisation: A Pulsed Semiconductor Parameter Analyser System for III-V FETs", *Journal of Electrical and Electronics Engineering, Australia*, vol. 14, no. 3, September 1994, pp. 196–205.

[6] Anthony E Parker and David J. Skellern, "Improved MESFET Characterisation for Analog Circuit Design and Analysis", *1992 International GaAs IC Symposium Technical Digest*, Miami Beach, October 4–7, 1994, pp225–228.

[7] A. E. Parker, "Implementing SPICE models with high-order continuity and rate dependence", *IEE Proceedings on Circuits, Devices and Systems*, vol. 141, no. 4, August 1994, pp251–257.

[8] Ron M. Kielkowski, *Inside SPICE: Overcoming the obstacles of circuit simulation*, McGraw-Hill, 1994.

[9] MicroSim Corporation, *The Design Centre, Analysis—User's Guide*, Irvine, CA 92718, USA.

[10] Jonathan B Scott and Anthony E Parker, "Distortion Analysis using SPICE" *Journal of the Audio Engineering Society*, vol. 43, no. 12, December 1995.

[11] Bashir Al-Hashimi, *The Art of Simulation using PSpice*, CRC Press, 1995.

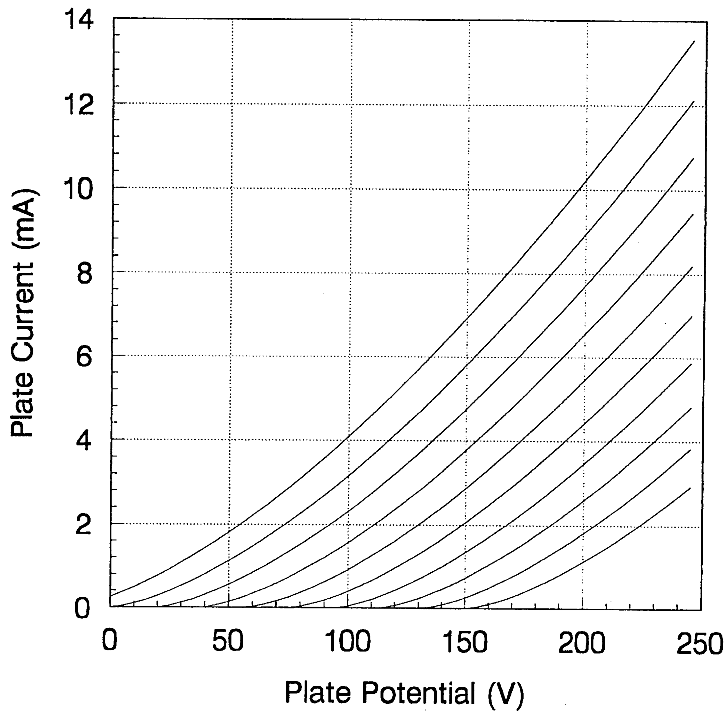

Figure 1: Plot of theoretical triode plate current curves against plate voltage for grid voltages from -2 to +0.25V in 0.25V steps.

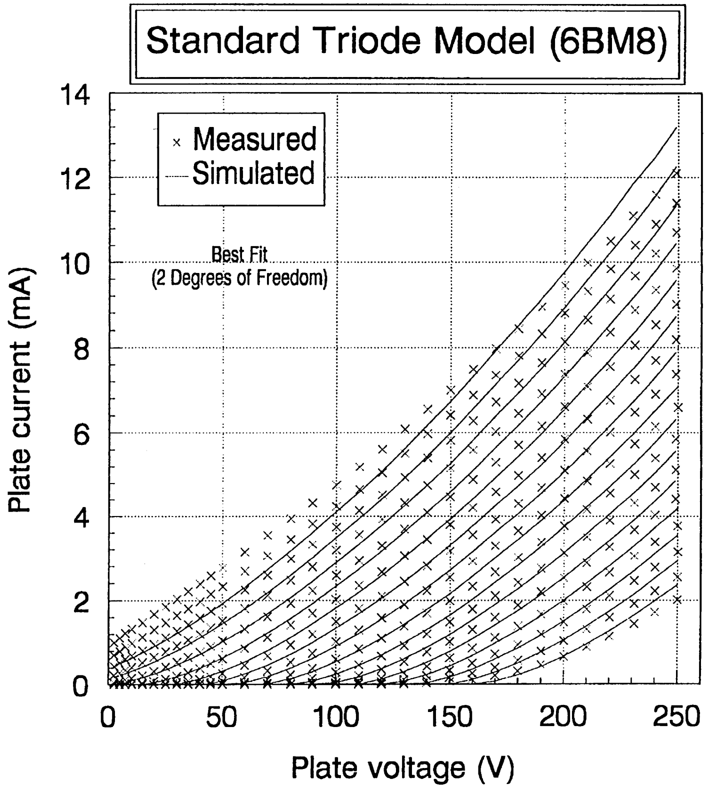

Figure 2: Characteristics measured and predicted with the standard triode equation, for the triode half of a 6BM8 valve.

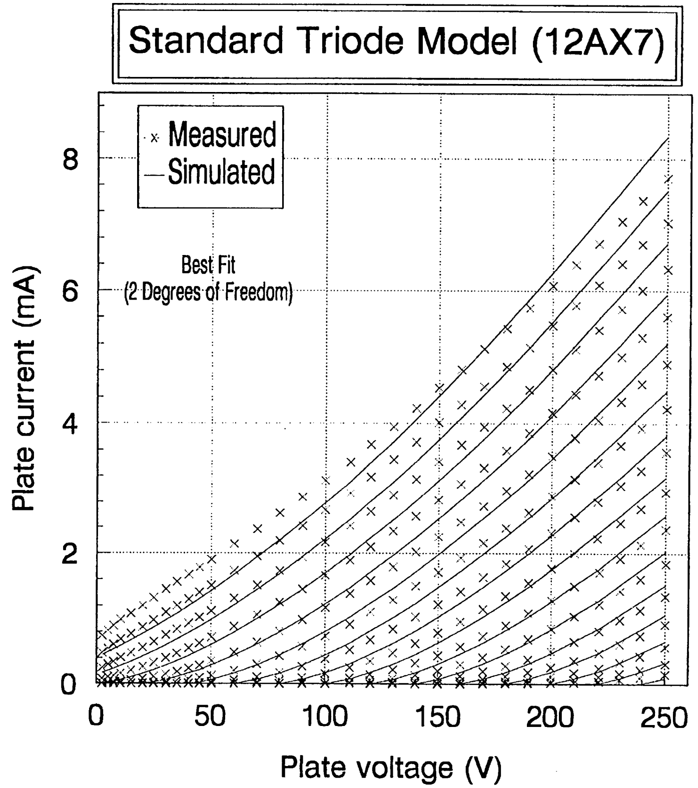

Figure 3: Characteristics measured and predicted with the standard triode equation, for a 12AX7 valve.

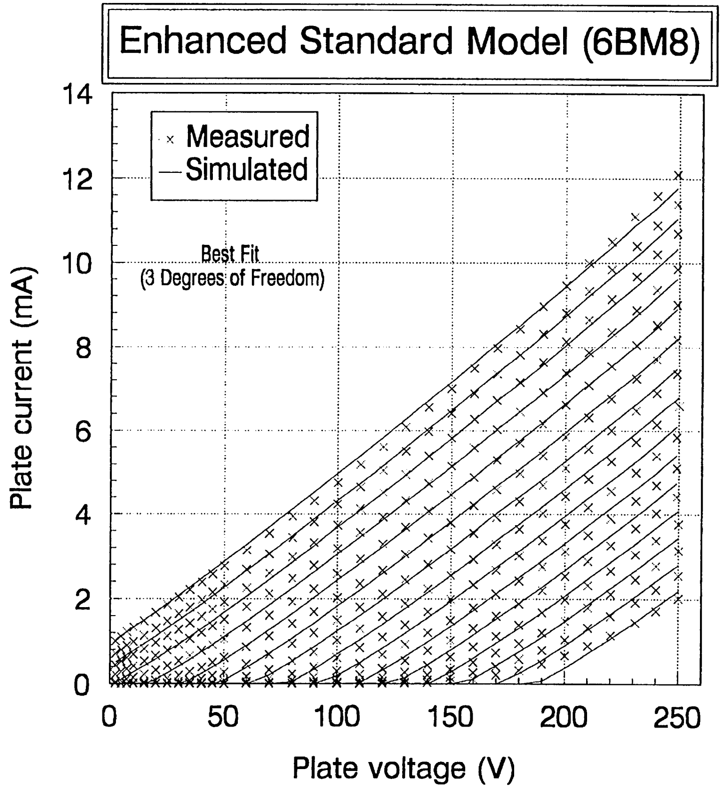

Figure 4: Characteristics measured and predicted with the triode equation using 3 degrees of freedom, for the triode half of a 6BM8 valve.

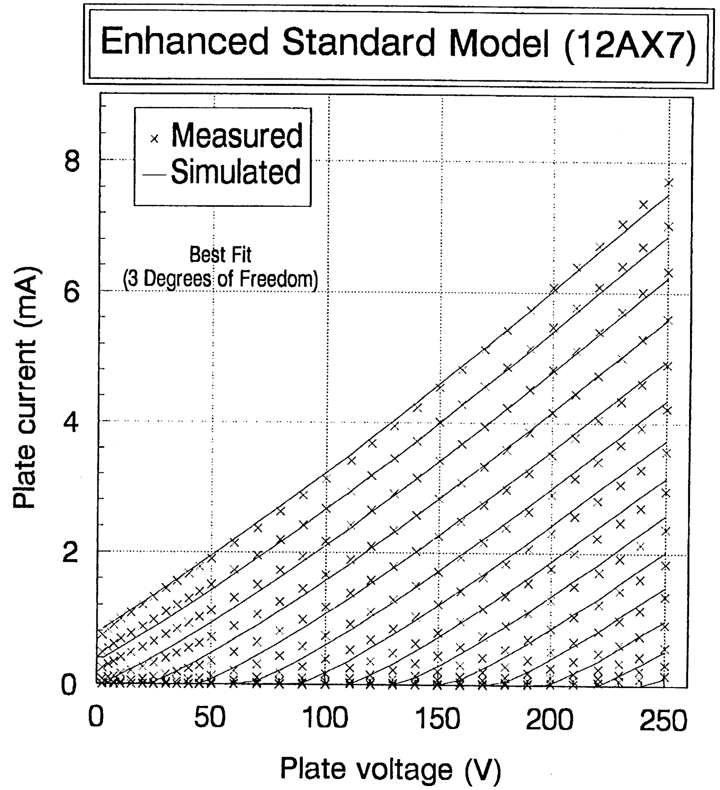

Figure 5: Characteristics measured and predicted with the triode equation using 3 degrees of freedom, for a 12AX7 valve.

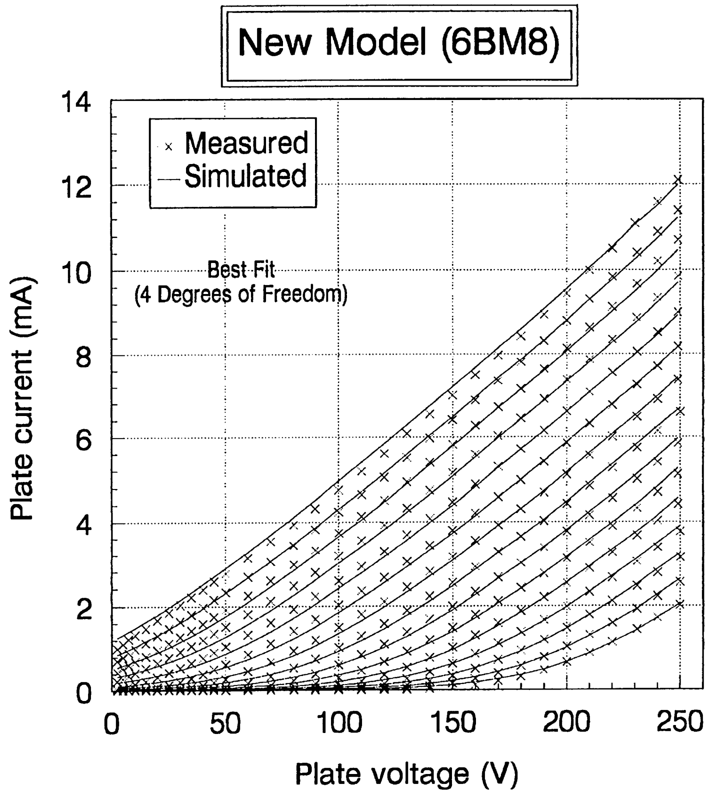

Figure 6: Characteristics measured and predicted with the new equation, for the triode half of a 6BM8 valve.

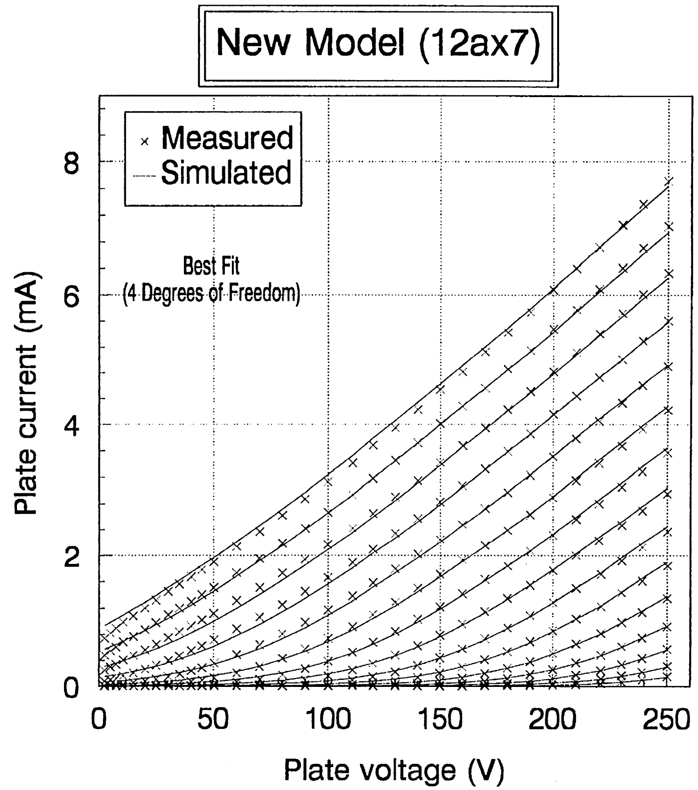

Figure 7: Characteristics measured and predicted with the new equation, for a 12AX7 valve.

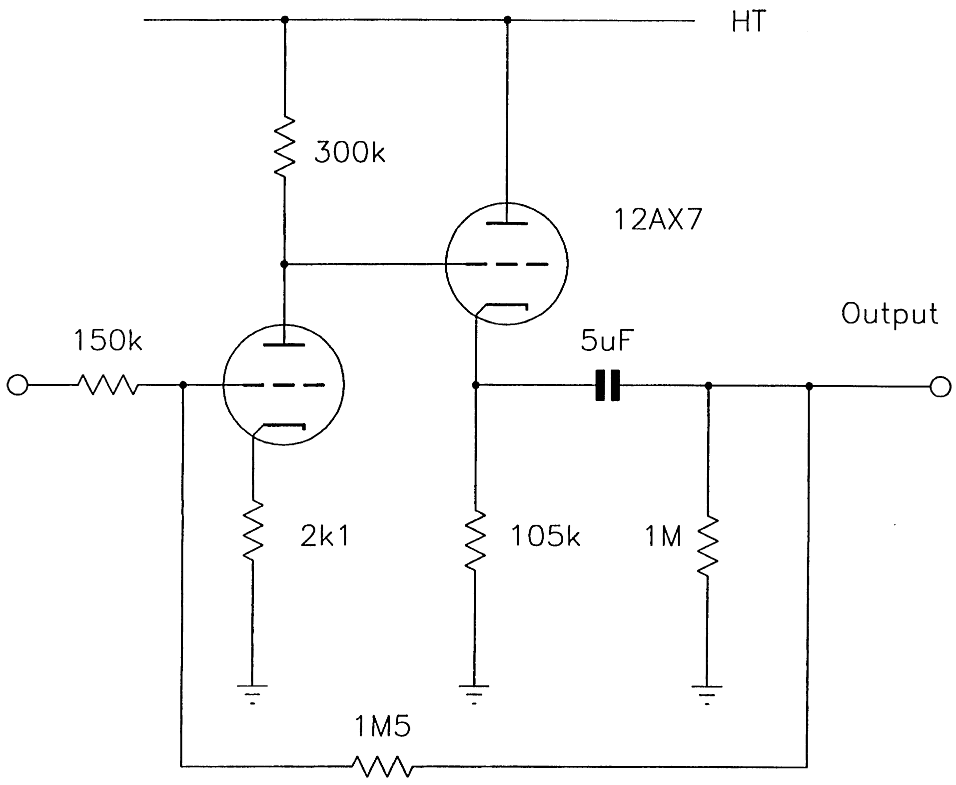

Figure 8: Triode preamplifier circuit using a 12AX7 twin-triode valve.

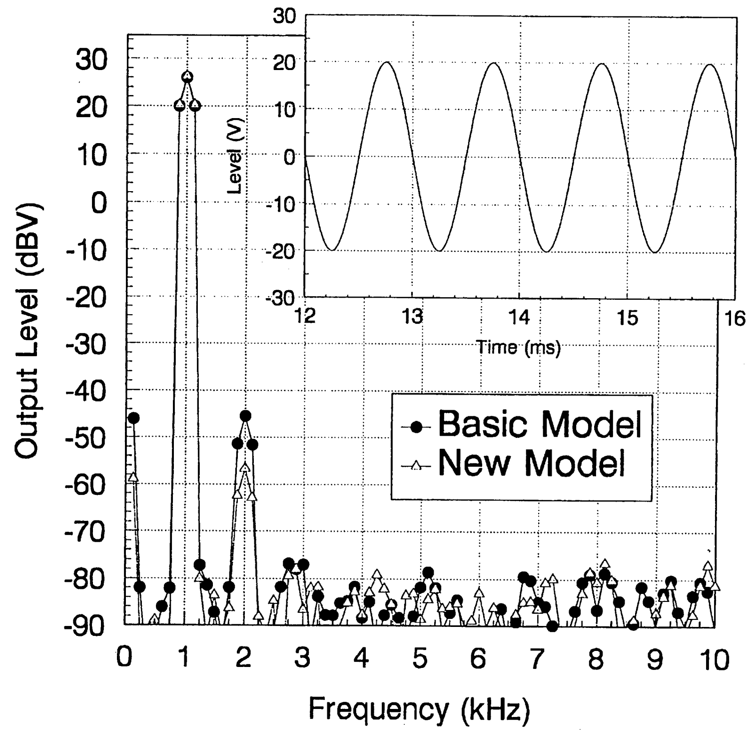

Figure 9: Predicted spectrum for the circuit of figure 8. The inset shows the output signal in the time domain.