Simulation of Electron Tubes with Spice.

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Simulation of Electron Tubes with Spice

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Abstract - Simulation of vacuum tubes is now a necessary technique in the design of a modern tube amplifier. This paper describes the design of simulation models that are as complet as possible in terms of reproduction of the Ip/Vp and screen grid curves, and in consequence the precision of simulation is very high. The distortion for instance, which is a good indicator of precision, shows a close correspondance with the measurements and the polarisation is very precise. A complete library including about 40 different tubes has been created, with which it should be possible to apply successfully the concept of virtual modelling to Hi-Fi or RF amplifiers, and also to historical research into the beginnings of radio communications.

1. Introduction

For 23 years, Spice has been the electronic simulator generally used. Spice is the acronym for Simulation Program with Integreted Circuit Enphasis. This means that Spice is an IC oriented programme. Up to now, there has not been any possibility of extending the fonctionnality of the software to use exotic components like electronic valves, triodes, tetrodes, pentode and so on. Nowadays, new commercial versions of this software have the possibility to integrate mathematical formulas inside. This means that there is a possibility to modelise this kind of component, if you know for instance, how to describe the inside relation between voltage and current . Moreover, precision in modellising is very important, because the results of a simulation are just as good is the basic premise.

The present work has been done with *Mathcad tm* to be used with the *ICAP Intosoft* version of *Spice 3e.1p*. Mathcad, like other mathematical softwares (like *Mathematica* for instance) is interesting in the way that it permits, without any programming, the implementation of non linear fitting. These sort of procedures are fundamental in adjusting parameters with very non linear equations. I must point out that the use of the two softwares are just indicative. You are not, in any way, obliged to use them to obtain results.

2. Limits of modeling

The aim is to modelise tubes like diode, triode tetrode and pentode for use in low frequency or if possible radio frequency up to, for instance, 30Mhz. We have exclude here complicated mixer tubes like Hexode, Octode, Nonode etc.. We exclude too for pertinency, gaze tubes like thyratron, regulator or Tungar's rectifier.

convergency) and covering of the function (which translates into terms of precision and capacity to assume marginal cases). That is why we must restrict the application perimetere to succeed. We have make the assumption that the valve in all cases is in the charge space regime and follow the 3/2 power Child-Langmuire law.

Second order parameters like the general variation of caracteristics with voltage filament or transitory regime when this arise are not covered. Noises generated in the tube and transit time in the RF context are not modelised now and should be the subject of further studies. In fact we have choosen to restrict the field to normal use which covers 99% of cases (this includes positive grid 1 and change in characteristics of a pentode with grid 2 potential).

Be carefule not to forget the large variation in parameters of this kind of components (about +/- 30%, but much less than the transistor Beta!). Its why we have chosen to modelise the valves from books or notices with "official" (thats means average) characteristics.

3. Principle

The method that we have employed consists of fitting, through a damped least square algorithme embedded in the software, a set of measured data with the pertinent expression to describe the relation between Ip and Vp of the device well. There are classical equations linking current and voltage in tubes. The more basic one is the diode equation:

$$\mathbf{Ip = g.V_a^{3/2}}, (1)$$

where Ia is the plate current, g the perveance of the diode and Va the anode or plate voltage. In fact the formula is not complete: one must add the contact voltage and the equation become:

$$\mathbf{Ip = g.(V_a+V_c)^{3/2}}. (2)$$

This formula is not difficult to implement in the computer, for instance to simulate vacuum rectifier. In this case, for instance, Vc is not important. But if one tries to simulate a radio detector system, it will be. The well known equation for the triode is just a bit more complicated. It is:

$$\mathbf{Ip = g.(Vg + Va/\mu)^{2/3}}, (3)$$

where Vg is the grid voltage and μ the amplification factor, the product of the internal resistance by the slope gm. In fact that expression leaves out two parasitic phenomenon existing in triodes : the change in gm and the change in the internal resistance when vg become more and more negative. It is why a new expression giving a better fit with the real caracteristics have been created for the sake of precision and completeness (see the next chapter). The new expression, emulate perfectly every kind of Triode tube, because there are enough parameters inside to fit each kind of non linearity. As for the triode, there is a very simple equation for the Pentode an Tetrodes, it is :

$$\mathbf{Ip = g.(Vg + Vg2/\mu g1g2 + Va/\mu)}, (4)$$

where Vg2 is the screen grid voltage, and $\mu g1g2$ is the gride one to screen grid amplification factor. This expression is right in the first instance if one leaves out the fact that the virtual cathodes tend to disappear when the anode voltage goes down before 50volts. like for the triode, in the chapter 5 a much more complete expression is suggested.

4. New expression for Triode

As we saw in the preceding paragraph, the expression leaves out two parasitic phenomenae existing in triodes : the change in gm and the change in the internal resistance when vg becomes more and more negative. That is why we have defined a new expresion giving a more realistic relation between Ip and Vp, as a function of Vg. In first we add the plate potential Vc, the formula become :

$$Ip = g.\left[Vg + \frac{Va + Vc}{\mu}\right]^{2/3}, \quad (5)$$

Then, by adding a coefficient B allowing to change g as a function of Vg. The expression become:

$$Ip = g.\left(1+\frac{Vg}{B}\right).\left[Vg + \frac{Va+Vc}{\mu}\right]^{2/3}, \quad (6)$$

when Vg is more negative, the left bracket become lower and the caracteristic mimic, as we will see, the real tube and tip over to the right. If we want to mimic the behavior for positive grid voltage too, the anode current must pass by zero and to do so, we must multiply the last formula by a coefficient depending on Va, with a form like Va/(Va+c) and the expression for the plate current becomes:

$$Ip = g.\left(1+\frac{Vg}{B}\right).\left[Vg + \frac{Va+Vc}{\mu}\right]^{3/2}.\left[\frac{Va}{Va+c}\right] \quad (7)$$

It is interesting to compare the different formulae, (3), (5), (6), (7) in terms of precision and capacity to fit real complicated curves. We have chosen the Telefunken tube **6463**, a double triode with a medium μ equal to 20 and S=5.2 mA/V, because its caracteristic is well defined for positive values of Vg. The results are visible on figure 1,2,3,4 where the squares represent data values.

As we can see, the figure 1 gives a rather rough fit, with a squared residuals coefficient **ERR**, of about 9.5. If one introduces Vc, which is the plate potential contact, the value of the residual figure become a little lower: 8.7 as we can see in fig2. Then, if one introduces the possibility to change the perveance factor g in relation to the value of Vg, the results become much better: 5.1. At least, its possible to fit very well the complete

caracteristics of that triode for positive grid voltage too, as we can see on figure 4 withe the formula (7). The error coefficient is just equal to 3.2 instead of 9.5, which is a three time factor improvement.

4.1 Improved formula for some peculiar kind of Triodes

There is a peculiar problem with the last expression. In fact if |Vg| is equal to the coefficient B, Ip become equal to zero. This is illustrated in figure 5 with a very non linear tube like the Russian **6C33C-B**. As one can see, not only is the global fit very bade (ERR=147), but for Vg= -140v, the slope of the current Ip is completely tilted and passes far from the datas points. That is why, another expression is given, for improved results with that peculiar type of triode:

$$Ip = g.\left(1+\frac{Vg}{B-\frac{Vg}{k}}\right).\left[Vg + \frac{Va+Vc}{\mu}\right]^{3/2} \quad (8)$$

The curves with the expression (8) give an ERR coefficient just equal to 40 instead of 147. It is a big improvement. As we can see, the curves for Vg=-100 and -140 volts pass near to the data points. This kind of expression should be used too for triodes like the **6080WA** or others serial regulator tubes, because they are in general very non linear.

5. Positive grid current.

Grid current for positive potential is a highly non reproduceable phenomenae. It depends of the technology (grid plating), temperature, vacuum, secondary emission, time and certainly plenty of other non evident parameters. However, for the sake of completeness, we have modellised the grid current through data given by Telefunken for the **6463**. As we will see in the next chapter, the general lok of the expression will be the same for the screen grid current of a Pentode or a Tetrode. The expression is the next one:

$$Ig1 = G.\left[\frac{A + Va}{B + Va}\right]^4 . Vg^{1.5} \quad (9)$$

The results are given on figure 7. The fit passes very well through the datas for Vg= +5v and +20v. For grid current with negative voltage, the curent is depending principally on vacuum and electrodes temperature. For the sake of simplicity, it is possible to just put a current generator in parallel with the grid-cathode space for the negative current and a vacuum diode for positive grid voltage. In lot of cases, that emulation is good enough.

6. Expression for a Pentode

The plate caracteristic of a Pentode is very different versus a Triode. In first, the Pentode is a current generator, this mean a very high internal resistance. Another caracteristic given by the existance of the suppressor grid is the rounded caracteristic under about 50v for Vp. It is why the expresion linking the plate voltage and the current voltage is very different relatively to the Triode. If we leave for a moment this last peculiarity, we can write :

$$Ip = g.\left[Vg + \frac{Vg2}{\mu g1g2} + \frac{Va}{\mu} \right]^{3/2} \qquad (10)$$

In that expression, μg1g2 is the amplification coefficient of the triode composed of the cathode, the command grid and the screen grid. Current values goes between 6 and 30. Mu is between one hundred and seven thousand. It is clear from (10) that the current does not depend from the anode voltage if μ is very high.

Now, if we want to put the rounded caracteristic in the model, we must introduce a function describing the fact that the virtual cathode disappears if Vp becomes lower than 50v. This is done by the coefficient k1. Next we must include a factor (k2) describing the fact that the internal resistance increase as the command grid becomes more and more negative. The coefficient k3 is fixed to 0.6 for low power penthode. This is a "cosmetic" coefficient, because without it, the characteristics do not pass trough zero for very negative grid one voltage. The coefficient μ is like in the triode, the global amplification factor.

$$Ia(Va, Vg) = G.\left[Vg + \frac{Vg2.\dfrac{Va - k3.Vg}{Va + \dfrac{Vg2}{k1}}}{\mu g1g2} + \frac{Va}{\mu.(1 - \dfrac{Vg}{k2})} \right]^{3/2} \qquad (11)$$

For an exemple, we will examine a low power Pentode, the **E80F**. This tube is very well specified by Telefunken. There is the folowing twelve value for Ip and Vp (Vg2=100v) :

Table 1

Vg	0	0	0	-1	-1	-1	-2	-2	-2	-4	-4	-4
Va	60	200	500	20	80	400	20	100	500	20	200	400
Ia	7.5	8.5	8.75	4.4	5.5	6	3	3.5	3.75	.5	.7	.7

To extract the modelising parameters, we have to know first the value of μ for Vg=0. To do that, we must calculate the slope of the internal resistance by ρ=ΔVa/ΔIa and the transconductance gm=ΔIa/ΔVg. in that case we have ρ=.9 Mégohm and gm=2.8 Ma/V. Therefore, μ must be made equal to about 2500. Now we can try to fit the complete characteristics of the Pentode with twelve equations for four unknown in terms of least square. The result in figure 8 was done withe μ=2800.

The resulting valure are: D (or k1) = 28.6, μg1g2= 21.6, C or k2= .167 and G= .749. Here, the model mimic the real tube quit well. The same expression apply very well to a power Pentode like the well known **EL84**, as we can see on figure 9. With a little quirk, it is possible to represent the steep variation of current under 50v from a beam Tetrode like the **6L6** or **KT88** as we will see in the next chapter.

7. Quirk peculiarity to simulate a Beam Tetrode

Just by adding a logical condition on the value of the plate current Ia fonction of Va and Vg in the expression (12) (which is is possible in Spice 3e2 version), it is possible to do a choise between (12) and (13). The (13) expression give the value of the plate current V_A when the virtual cathode have diseapered, for instance under 50v.

$$I_a(V_a, V_g) := .924 \cdot \left[V_g + \frac{V_{g2}}{7.71} \cdot \frac{V_a}{V_a + \dfrac{V_{g2}}{200}} + \frac{V_a}{140 \cdot \left(1 - \dfrac{V_g}{4}\right)} \right]^{1.5}$$

$$I_A(V_a) := .02 \cdot V_a^{2.5}$$

$$I_c(V_a, V_g) := \mathrm{if}\left(I_A(V_a) < I_a(V_a, V_g), I_A(V_a), I_a(V_a, V_g)\right) \qquad (12, 13, 14)$$

The last Mathcad expression (14) means : if the current given by the expression (13) is lower than the current given by (12), then the current is given by (13) else by (12). It is possible too to implement the same logical statement in Spice and obtain juste the same results in simulation. The expression (13) is an experimental one. Note the formal similarity whis the expression of the vacuum diode. One can see the 6L6 Ip/Vp characteristics on the figure 10. On the top at left, the "Penthode" characteristics, without the expression (13).

8. Expression to simulate the screen grid current

In chapter 5 we have seen an expression giving the grid current of a Triode. Here, the expression is almost the same: one have just to insert the screen grid voltage and the value of the exponent. The expression is the following:

$$Ig2 = G.\left[\frac{k1+Va}{k2+Va}\right]^3.\left[Vg + \frac{Vg2}{k3}\right]^{1.5} \qquad (15)$$

In general, the ratio between k1 and k2 is near 2. For the **EL84**, k1=22, k2=11.7 and k3= 17.8. The results are visible on figure 11. For the sake of quality of the expression, Vg1 and Vg2 are different for the three curves. In the three case the curve pass near to the data points, which is the proof of the goodness of the expression.

9. Comparaison between model and data in termes of polarisation, banwith and distorsion

There is not now a complete set of data comparing a given number of models and real tubes. However, as you know, it can existe big differences between tubes in a same brand and much more between different brands. It is why we just did a first mesurement whis an E803S from TELEFUNKEN, which is a bettered version of the ECC83.

The power supplie was 200v, the plate resistor had a value of 47K, the cathode resistor was 1.5K decoupled by a capacitor of 50µF. Then the plate was loaded by a resistor of 150K, through a capacitor of 0.1µ .

The bandwith depend of the lay-out and the internal capacitances of the tube. As a linear phenomenon, there is any difficulty to fit the value to have good results. Be carefull, because the divert capacitance change when the tube is hot. The polarisation was very near the reality, at about some tenth of millivolts. The mesured distortion of each triodes where around 7 to 8% (with some noise from main). With the simulation, the Fourier transforme gave 5.8%. The .CKT is at the end of the paper, in the annexe.

10. Conclusions

Up to now, there was no practical possibility to use tubes in Spice, in particular Pentodes and beam Tetrodes. For the first time we have define a complete set of new and more precise expressions than in classical textbooks. A methode to fit the different parameters, like plate and the divert grid currents, is given too.

Results in termes of plate or grid current aggree very well as we saw. The validation in terme of distortion is less evident because there is a big variability in caracteristics due to construction and evolution in time of caracteristics. We have to work more on that interresting and practical aspect of validation. We hope to publish anothere paper on it in a few month.

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12. Annexe

Here, the .CKT, which is the .CIR+ the SUBCKT.

```
C:\SPICE4\CIRCUITS\TUBES3\ECC83
*SPICE_NET
*INCLUDE LAMPES.LIB
.SUBCKT TRIO2#0 A G C
*
B1 A C I=1.0200M*((1+V(G,C)/18.000 )*(V(G,C)+((V(A,C)+50.700 )/90.600 ))^1.5)*
+(V(A,C)/(V(A,C)+5.0000 ))
.ENDS
.SUBCKT IGRID#0 G1 C
* COURANT GRILLE
B4 6 0 V=V(G1,C)>0 ?10.0000U*V(G1,C)^1.5 : 100.000N/-(V(G1,C)-.1)
B5 G1 C I=V(6,0)
.ENDS
.SUBCKT ECC83  1 2 3
*       ANODE G CAT
*
X1 1 2 3 TRIO2#0  ........................................ passing parameters
*{G=1.02M MU=90.6 B=18 VC=50.7 K=5}
X2 2 3 IGRID#0
*{ALPHA=.01M BETA=.1U} ........................ passing parameters

C1 1 2  1.7PF
C2 2 3  1.6PF .................................................. Tree parasitics capacitors
C3 1 3  .46PF
*
.ENDS
.TRAN 10U 10M 0 5U  ................................. 10 ms to simulate with 10us inc.
.FOUR 1000HZ V(0,4)  ................................. Calculate Fourier transform
*ALIAS  V(4)=OUT
.PRINT TRAN  V(4)
X1 1 3 2 ECC83
R2 2 0 1.5K .................................................. Cathode resistor
R3 3 0 1MEG  ................................................. Grid resistor
```

C1 3 6 10N
C2 1 4 .1U
R4 4 0 150K .... **Load resistor**
V1 5 0 200V
V2 6 0 SIN 0 .8 1K .... **.8v crête at 1Khz**
C3 2 0 50U .... **Decoupling capacitor**
R1 1 5 47K .... **Plate resistor**
.END

$$I_a(V_a, V_g) := 1.625 \cdot \left(V_g + \frac{V_a}{18.13}\right)^{1.5}$$

ERR = 9.476

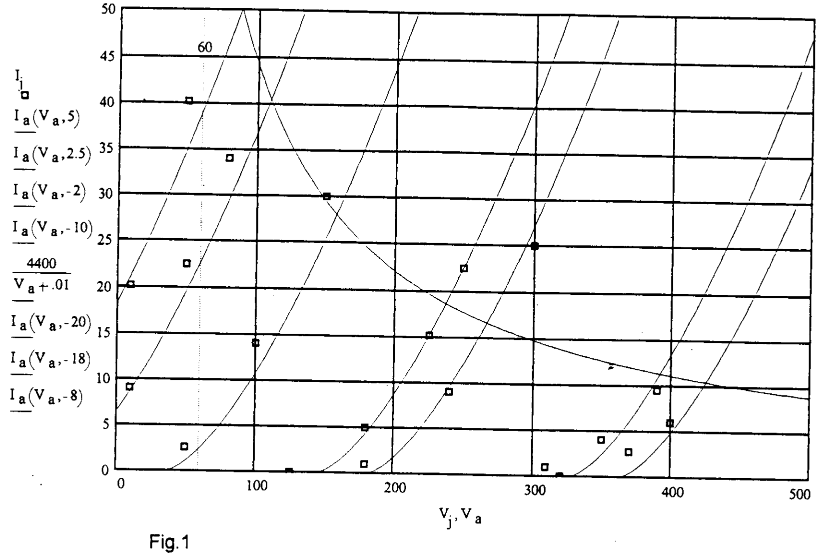

Fig.1

$$I_a(V_a, V_g) := 1.445 \cdot \left(V_g + \frac{V_a + 17.2}{18.5}\right)^{1.5}$$

ERR = 8.727

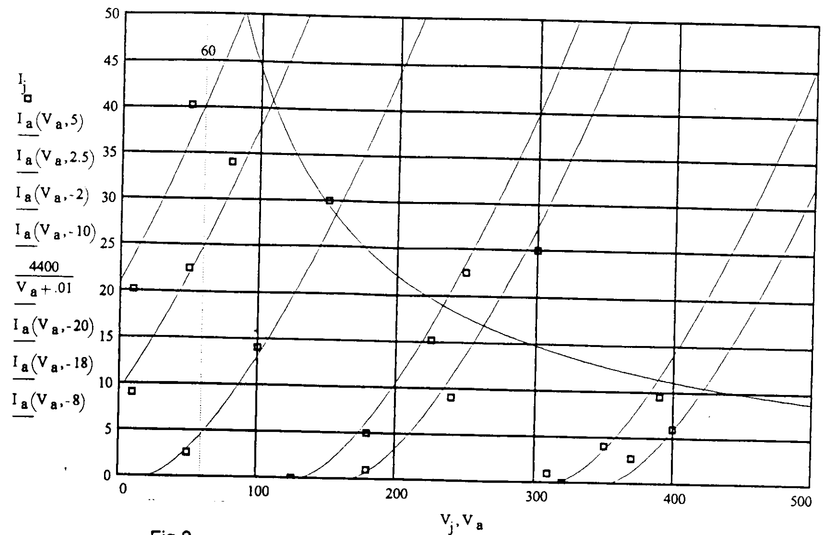

Fig.2

$$I_a(V_a, V_g) := \left(1 + \frac{V_g}{26.73}\right) \cdot 1.451 \cdot \left(V_g + \frac{V_a - 1.08}{15.5}\right)^{1.5} \quad ERR = 5.109$$

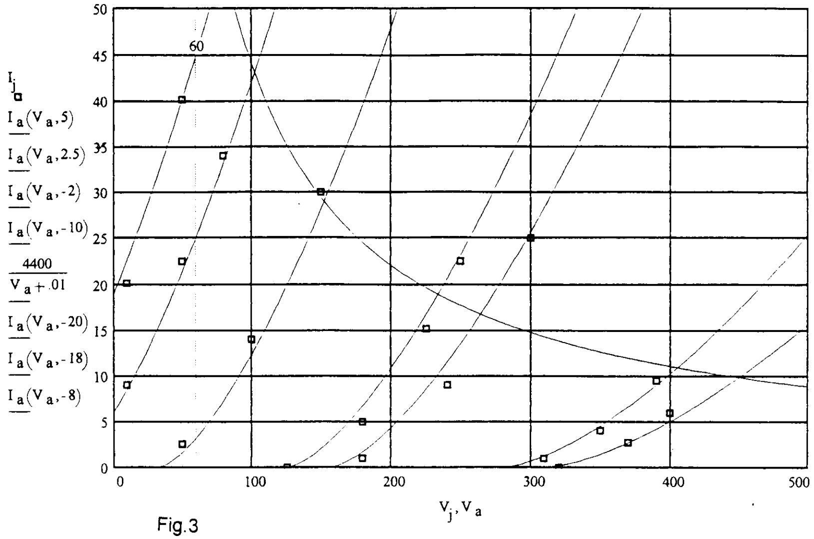

Fig.3

$$I_a(V_a, V_g) := \left(1 + \frac{V_g}{34.8}\right) \cdot 1.51 \cdot \left(V_g + \frac{V_a + 11.9}{17}\right)^{1.5} \cdot \frac{V_a}{V_a + 3.76} \quad ERR = 3.253$$

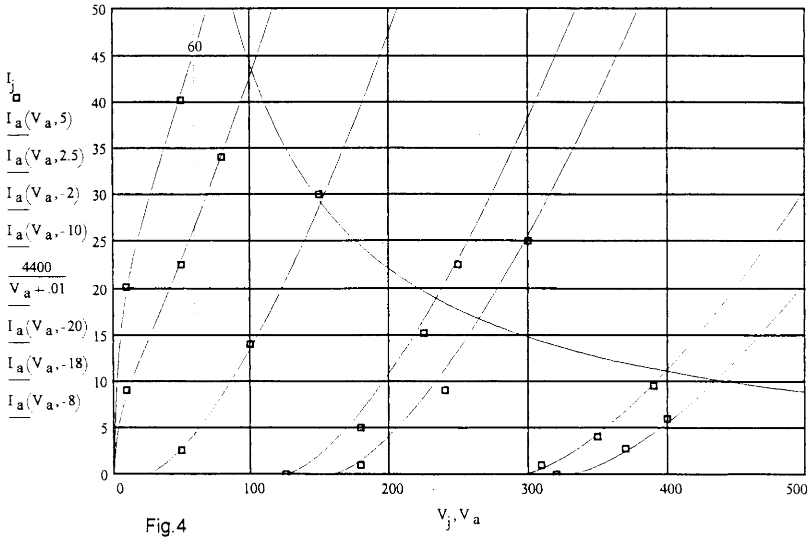

Fig.4

$$I_a(V_a, V_g) := \left(1 + \frac{V_g}{154}\right) \cdot 2.89 \cdot \left(V_g + \frac{V_a + 16.5}{2.244}\right)^{1.5} \quad ERR = 147.114$$

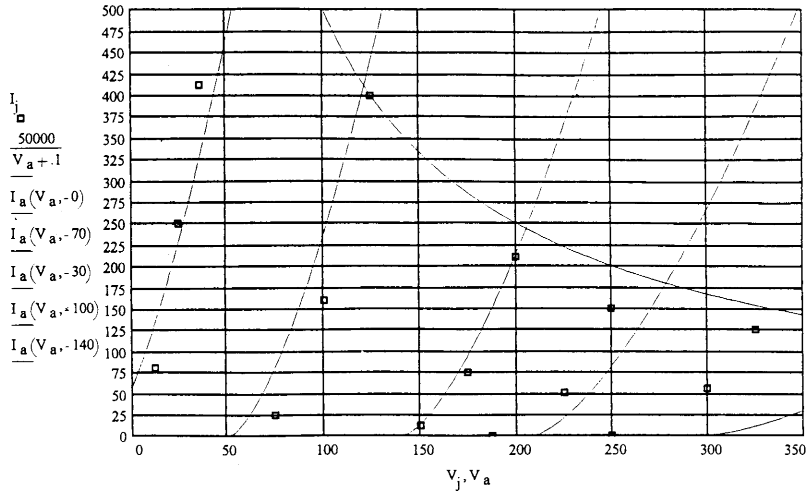

Fig.5

$$I_a(V_a, V_g) := \left[1 + \frac{V_g}{16.976 - \frac{V_g}{.941}}\right] \cdot 5.457 \cdot \left(V_g + \frac{V_a - .578}{1.984}\right)^{1.5}$$

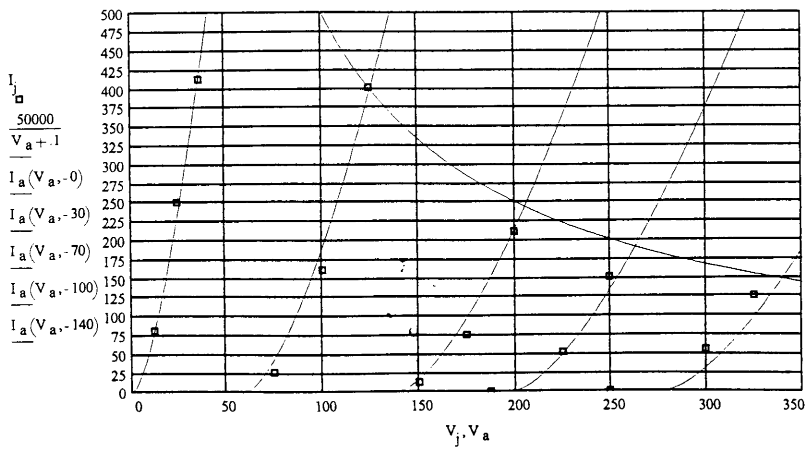

Fig.6

$$MINERR(g,K1,K2) = \begin{pmatrix} 0.598 \\ 26.44 \\ 21.573 \end{pmatrix} \qquad ERR = 1.513$$

$$V_a := 0, 3 .. 150 \qquad I_g(V_a, V_g) := .6 \cdot \left(\frac{26.44 + V_a}{21.57 + V_a}\right)^4 \cdot V_g^{1.5}$$

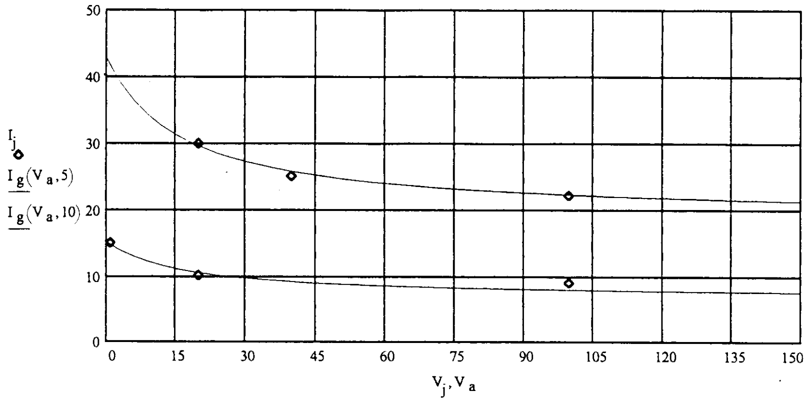

Fig.7

$$I_a(V_a, V_g) := G \cdot \left[V_g + \frac{V_{g2}}{\mu_{g1g2}} \cdot \left[\frac{V_a - .6 \cdot V_g}{V_a + \frac{V_{g2}}{D}} \right] + \frac{V_a}{\mu \cdot \left(1 - \frac{V_g}{C}\right)} \right]^{\frac{3}{2}}$$

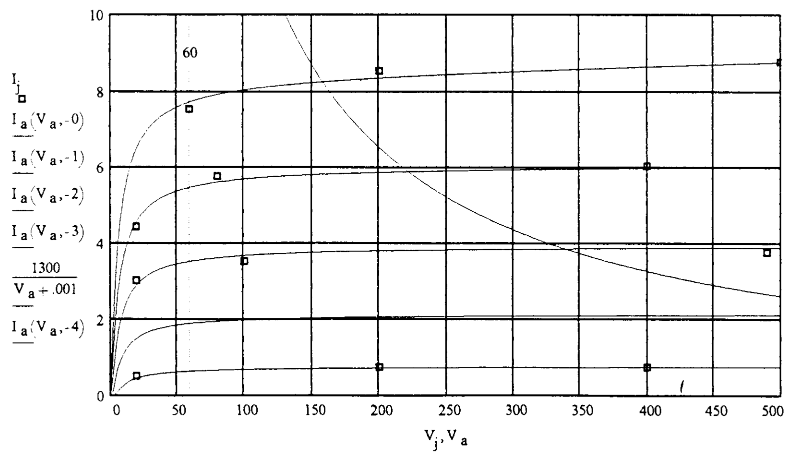

Fig.8 E80F, Vg2=100v

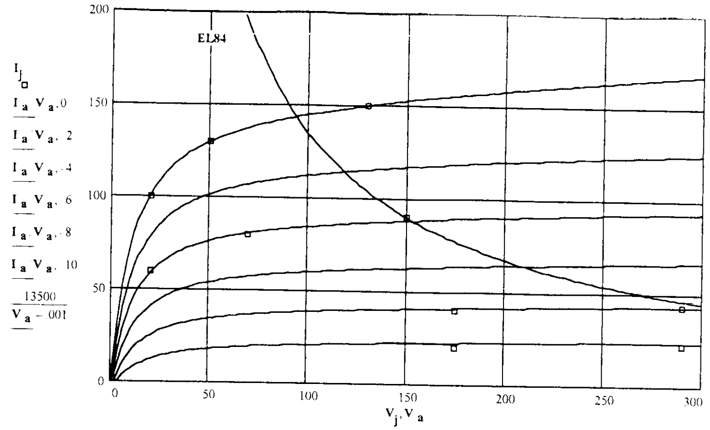

Figure 9

$$I_{g2}(V_a, V_g, V_{g2}) := .219 \cdot \left(\frac{V_a - 22}{V_a - 11.7}\right)^3 \cdot \left(V_g - \frac{V_{g2}}{17.8}\right)^{1.5}$$

EL84_Ig2 := 250

$j := 1 .. 10$

V_j :=	I_j :=
0	21
50	4.6
100	3.9
250	3.1
0	10
100	2.3
250	1.9
0	8.2
70	2
150	1.7

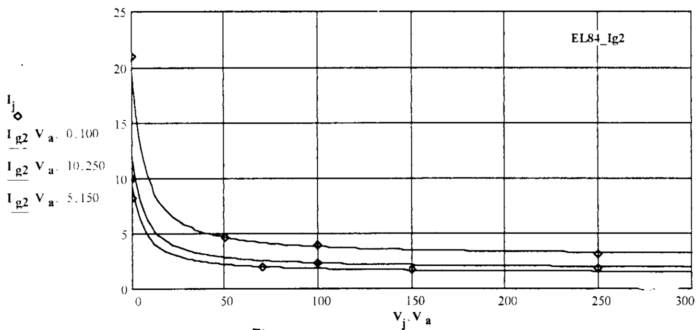

Fig.10

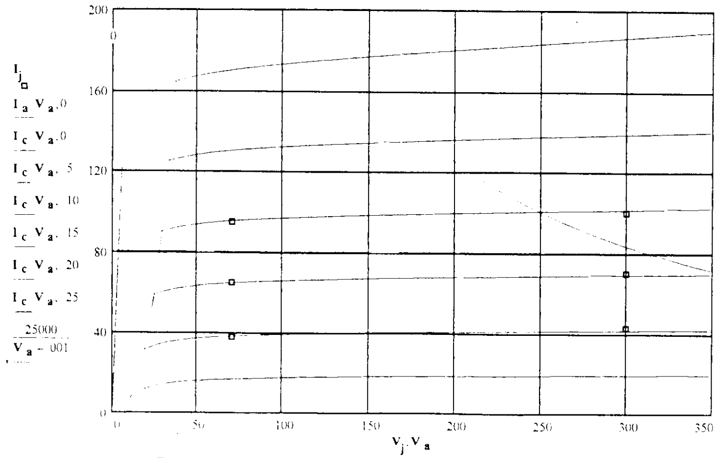

Fig 11