Abstract — Coupled magnetics filter techniques are important tools in a converter designer’s arsenal, but are not well understood. Evidence of this is one basic building block of coupled filters, reinvented numerous times during the past 65 years. A detailed analysis of coupled magnetics for dc-dc converters, based on a key building block, is given.

I. INTRODUCTION

Coupled-inductor and other integrated-magnetics techniques have existed for many years, but most power electronics engineers are uncomfortable with them. This may be because of limited experience with coupled magnetics. Or it could be explained by the level of complexity in many treatments of coupled magnetic devices. Unlike most networks, coupled-inductor techniques involve simultaneous parallel energy-transfer pathways: electrical and magnetic. Despite the difficulty, the circuits are useful and deserve to be better appreciated.

The technique described in this paper, shown in Fig. 1, replaces a series smoothing choke with a “smoothing transformer” (a pair of coupled inductors) and a blocking capacitor. Because these components form a linear two-port filter, the technique can be applied to any dc circuit, and reduces the ripple current wherever a choke is currently used. Thus it may be applied to the dc input of a converter, its dc output, or an internal dc link (in applications such as motor drives or HVDC transmission).

The concept to be described is not a new one. It has surfaced in many forms over the years, but it nonetheless appears to be little known, and is not well understood. The purpose of this paper is fourfold:

• to bring the technique to the attention of a wider audience of power electronics designers;

• to give a simple explanation of its operation, avoiding magnetics theory;

• to show how it can be usefully applied in practice;

• to give an indication of broader possibilities for coupled magnetics.

II. PRINCIPLE OF OPERATION

As shown in Fig. 1, the filter has two ports. It is rather unhelpful to think of these as inputs or outputs, because the power flow may be in either direction. Instead, we label one port “noisy” (connected to the ripple source), the other “quiet”. The aim is to produce a low ac current (ideally zero) at the quiet port. Further, it can be misleading to think of the smoothing transformer’s windings as a primary and secondary. Hence we label them the “dc” and “ac” windings to indicate their purpose in the circuit. The dc winding carries a heavy direct current (like a smoothing choke), while the ac winding carries only a small ac ripple current.

A. Infinite Capacitor, Perfect Coupling

Suppose the transformer is ideal, with a one-to-one ratio, and that the dc blocking capacitor $C$ is infinite. Let the voltage at the noisy port consist of a dc component $V_n$ and a superimposed ac ripple $V_{n~}$:

$$V_n = V_n + V_{n~}.$$  

Because $C$ is infinite, the voltage across it has no ac component: it is just $V_n$. Applying KVL, the voltage across the transformer’s ac winding must be $V_{n~}$. This voltage is transferred unchanged to the transformer’s dc winding. Again by KVL, the voltage at the quiet port must be $V_q = V_n - V_{n~}$. So the quiet port sees only the dc component of the voltage at the noisy port: ideal ripple filtering, with $V_q = 0$.

Essentially the infinite blocking capacitor ensures that the transformer’s ac winding picks up the whole of the ac ripple voltage. By ideal transformer action this voltage is applied in series with the dc winding in opposition to the original ripple voltage. Thus the quiet port sees pure, ripple-free dc.

B. Infinite Capacitor, Imperfect Coupling

Of course, an ideal transformer is not physically realizable, so the assumptions need to be relaxed somewhat. Equivalent circuits (as in Fig. 2) support detailed analysis. Suppose the
dc winding comprises \(N_{dc}\) turns and has self-inductance \(L_{dc}\); and the ac winding has \(N_{ac}\) turns and self-inductance \(L_{ac}\). Let the mutual inductance of the pair of windings be \(M\). We can also employ the winding coupling coefficient \(k\), defined by

\[
k = \frac{M}{\sqrt{L_{dc}L_{ac}}}
\]

where \(0 < k < 1\). The transformer can be replaced by its conventional T-equivalent circuit, shown in Fig. 2 (with an extra output capacitor), in which the three inductances are

\[
L_A = M = k\sqrt{L_{ac}L_{dc}} \\
L_B = L_{ac} - M = L_{ac} - k\sqrt{L_{ac}L_{dc}} \\
L_C = L_{dc} - M = L_{dc} - k\sqrt{L_{ac}L_{dc}}
\]

Retaining the assumption that \(C\) is infinite, ignoring \(C_2\), and considering the \(L_A-L_B\) voltage divider, substituting from (2) we see that the ripple gain of the circuit is

\[
\frac{V_q}{V_n} = \frac{L_B}{L_A + L_B} = 1 - k\sqrt{\frac{L_{dc}}{L_{ac}}}
\]

(We have dropped the “~” for the ac components, but it should be understood throughout the remainder of the paper.) From (3), the condition for zero ripple at the quiet port is

\[
\text{Null condition: } k = \sqrt{\frac{L_{ac}}{L_{dc}}}
\]

Since \(k < 1\) in practice, \(L_{ac} < L_{dc}\): the ac winding will have fewer turns than the dc winding.

### C. Remarks

1. Because it relies on cancellation, the null condition is very sensitive to \(k\). However, if the condition is not met exactly, there can still be considerable ripple attenuation. For example, if there is a 1\% mismatch so that \(k = 0.99\sqrt{(L_{ac}/L_{dc})}\), (3) shows that the ripple gain is \(-40\)dB. Similarly, a 10\% mismatch gives \(-20\)dB.

2. A special case arises when \(k = \sqrt{(L_{ac}/L_{dc})} = 1/\sqrt{2}\). Then the equivalent circuit has \(L_A = L_C = L_{ac}/2\). Because of this symmetry the distinction between noisy and quiet ports vanishes, and the circuit filters ripple equally in either direction.

3. It is important to understand that the ripple is truly zero only in the limiting case of infinite blocking capacitance. With finite capacitance, the circuit becomes a low-pass filter.

4. In principle, the smoothing transformer need be only slightly larger than an equivalent choke \(L_{dc}\). This is because the ac winding carries only the ripple current, and can use much thinner wire than the dc winding.

### III. HISTORY AND REINVENTION

Coupled magnetic devices such as Fig. 1 were more frequently encountered in the early days of rectification. The circuit appears in a complete form by 1930 [1], [2], and a variety of related circuits were also known. Writing in 1945, Turney [3] gives one the impression that coupled inductors were at one time relatively routine extensions of chokes; his book shows several combinations of inductor windings that would be suitable for Fig. 1, and shows how to analyze them as uncoupled equivalents. He merely suggests that the coupled combinations are “very useful in designing networks,” and hints at the possible advantages for filter applications.

More recently, there have been several instances of “invention” of the circuit in Fig. 1. Tapped winding versions equivalent to Fig. 1 appear in a 1947 text [4] and a 1957 handbook [5]. Ten years later, Lloyd [6] presented the circuit in a rectifier context and, unable to locate any older references, he boldly claimed to have invented coupled-magnetics filtering. The circuit was subsequently presented by Feng et al. [7] in 1970, along with detailed analysis, as an alternative to active ripple reduction. It reappeared in the 1970s and 80s: Rensink [8] studied circuits similar to those contained in [2]. Scopes [8] included coupled-inductor filtering in his book on rectifiers, and in 1985 Severns and Bloom [10] discussed several ripple cancellation circuits, including Fig. 1, in the context of dc-dc converters.

The 1990s have seen an explosion of reinvention. Marrero [11–13] managed to obtain a patent on applications to forward converters although his reference list goes back to [1]. In 1996 a New Zealand group [14] re-introduced the “smoothing transformer” (the circuit of Fig. 1) as a “new concept” for an HVDC application. Dc-dc converter applications were discussed at about the same time by Canadian workers [15], apparently unaware of the Severns–Bloom work. An excellent 1997 discussion of ripple cancellation and coupling concepts can be found in [17], which also includes Fig. 1. The circuit continues to be reinvented, the latest case that has come to our notice being in 1998 [18].

Aside from the issue of repeated reinvention, it is disappointing that the more recent power electronics work seems to treat the smoothing circuit of Fig. 1 as an integral part of the converter, missing the point that it is a linear two-port which can be analyzed independently and added to a wide variety of circuits. This points to a need for analysis and modeling, to allow the routine application of such circuits.
IV. ANALYSIS AND SIMULATION

We now examine the effects of practical limitations on the performance of the smoothing circuit. For clarity, our exposition proceeds by adding imperfections one at a time.

A. Finite Blocking Capacitance

If the dc blocking capacitance in Fig. 1 is finite with value \( C \), the transfer function of the filter is

\[
\frac{V_q}{V_n} = \frac{1}{1 + s^2 L_{ac} C (1 - k \sqrt{L_{dc}/L_{ac}})}
\]  

(5)

At low frequencies the gain is unity, and at high frequencies it is \( 1 - k \sqrt{L_{dc}/L_{ac}} \). Several operating modes are of interest.

B. Second-Order Low-Pass Mode

When the null condition of (4) is imposed to reduce the infinite-frequency gain to zero, we obtain

\[
\frac{V_q}{V_n} = \frac{1}{1 + s^2 L_{ac} C}
\]  

(6)

This is a second-order low pass filter. In fact, (6) is identical to the transfer function of a simple low-pass filter formed from an inductance \( L_{ac} \) and a capacitance \( C \). The cutoff frequency is \( \omega_0 = 1/L_{ac} C \). Compare this to a simple low-pass filter with a second capacitor to ground. If the existing capacitor from (9) is \( C_1 \) and the additional capacitor is \( C_2 \), the transfer function with \( k = \sqrt{L_{ac}/L_{dc}} \) becomes

\[
\frac{V_q}{V_n} = \frac{1}{1 + s^2 (L_{ac} C_1 + L_{dc} C_2) + s^4 C_1 C_2 L_{ac} (L_{dc} C - L_{ac})}
\]  

(7)

This is a fourth-order low-pass filter, and the high-frequency gain falls by 80dB/decade. This shows the true benefit of the smoothing transformer: its ability to integrate more than one inductance into a single magnetic structure, in this case making a fourth-order filter from three components.

In the special case of \( k = \sqrt{L_{ac}/L_{dc}} = 1/\sqrt{2} \) and \( C_1 = C \), the equivalent ladder-filter values are \( L_{dc}/2 \), \( C \), \( L_{dc}/2 \), \( C \), which gives the transfer function

\[
\frac{V_q}{V_n} = \frac{1}{1 + \frac{3}{2} s^2 L_{dc} C + \frac{1}{4} s^4 L_{dc}^2 C^2}
\]  

(8)

As an example, let \( L_{ac} = 50 \mu \text{H} \), \( L_{dc} = 100 \mu \text{H} \), \( k = 1/\sqrt{2} \) and \( C_1 = C_2 = 100 \mu \text{F} \). Each circuit element \( (L_{ac}, L_{dc}, C_1, C_2) \) has a parasitic series resistance of 100m\Omega. Fig. 3 shows the frequency response obtained using PSPICE. For comparison, a simple low-pass filter is shown, formed from \( L_{dc} = 100 \mu \text{H} \) and \( C_1 = C_2 = 100 \mu \text{F} \) (in parallel), each component again with series resistance of 100m\Omega.

D. Second-Order Notch Mode

The null condition (4) is derived from the infinite-C case, but when \( C \) is finite it is not the only possibility. Consider the numerator of (5). The gain becomes zero at \( \omega = \omega_1 \), where

\[
\omega_1 = \frac{1}{\sqrt{1 - k \sqrt{L_{dc}/L_{ac}} L_{ac} C}}
\]  

(9)

Thus by making \( k < \sqrt{L_{ac}/L_{dc}} \), it is possible to eliminate a single ripple frequency \( \omega_1 \). Referring to the equivalent circuit of Fig. 2, under these conditions \( L_B \) resonates with \( C \), reducing the vertical leg of the T to a short circuit at \( \omega_1 \) and giving a zero of transmission at this frequency.

The notch mode of operation is described in [14]. Unfortunately, equation (2) of [14] is incorrect — the open-circuit voltage ratio of the smoothing transformer is not equal to the turns ratio, as stated — so the paper’s results are invalid.

The value of \( k \sqrt{L_{dc}/L_{ac}} \) must not be too close to unity, or the notch frequency will be extremely sensitive to changes in \( k \), \( L_{dc} \) and \( L_{ac} \). On the other hand, \( k \sqrt{L_{dc}/L_{ac}} \) should be close to unity for good attenuation of frequencies above \( \omega_1 \), as the asymptotic high frequency gain is \( 1 - k \sqrt{L_{dc}/L_{ac}} \). In a practical design, a compromise is required. When \( k = \sqrt{L_{ac}/L_{dc}} \), \( \omega_1 \) moves to infinity, giving the low-pass mode. If \( k > \sqrt{L_{ac}/L_{dc}} \) there is no real value of \( \omega_1 \) and no notch appears.

E. Fourth-Order Notch Mode

As with the low-pass mode, it is sensible to terminate the filter with a second capacitor to ground. If the resonant capacitor from (9) is \( C_1 \) and the additional capacitor is \( C_2 \), the transfer function becomes

Figure 3. PSPICE result for coupled filter with \( k = 0.707 \).
\[
\frac{V_q}{V_n} = \frac{1 + s^2 \left(1 - k \frac{L_{dc}}{L_{ac}}\right) L_{ac} C_1}{1 + s^2 \left(L_{ac} C_1 + L_{dc} C_2\right) + s^4 \left(1 - k^2\right) C_1 C_2 L_{ac} L_{dc}}
\]  

The notch frequency \( \omega_1 \) is unchanged, but the gain now falls by 40dB/decade at high frequencies, instead of flattening out at \( 1 - k \sqrt{L_{dc}/L_{ac}} \).

Fig. 4 shows a simulation with \( L_{ac} = 50\mu\text{H}, L_{dc} = 100\mu\text{H}, C_2 = 100\mu\text{F} \), but reducing \( k \) to 0.568 and \( C_1 \) to 1\mu\text{F}, placing a notch at 50kHz. Each component again has a series resistance of 100m\text{\Omega}.

F. Winding Capacitances

Next, we consider the effects of non-zero winding capacitance. Suppose the dc winding has self-capacitance \( C_{dc} \), the ac winding has self-capacitance \( C_{ac} \), and there is an interwinding capacitance \( C_{iw} \). It turns out that only one of these, \( C_{dc} \), is important.

Consider first the ac winding self-capacitance, \( C_{ac} \). This forms a path for ac to flow from the quiet port to ground via the blocking capacitor, \( C \). But \( C \) is already passing the full ripple current, so the extra current from \( C_{ac} \) will make little difference to its voltage. Similarly, the interwinding capacitance forms a path from the quiet port to \( C \), but this makes it an effective load capacitance at the quiet port, much smaller than any added quiet port capacitance.

Finally, the dc winding’s self-capacitance \( C_{dc} \) bypasses the filter and interacts with it. This can be analyzed, and actually leads to a condition that allows a second notch to be added to the filter. When bypass capacitance \( C_3 \) (a combination of \( C_{dc} \) plus any capacitance added for tuning) appears across the filter, it can be shown that a transmission zero occurs at a single frequency, \( \omega = \omega_2 \), producing a notch at \( \omega_2 \). The required value of \( C_3 \) is:

\[
C_3 = \frac{1}{\omega_2^2 \left[L_{dc} - \omega_2^2 L_{ac} C_1 (L_{dc} - L_{ac})\right]}
\]  

Note that this is a different mechanism to the one that produced the notch at \( \omega_1 \) in (9).

The two notch mechanisms may be combined to provide a filter with a double notch. Abandoning the null condition of (4), we can adjust the parameter values to produce notches at two frequencies \( \omega_1 \) and \( \omega_2 \). The detail is not given here because of space limitations.

G. Effect of Parasitic Resistances

In a practical circuit, the windings of the transformer will have copper loss and the capacitors will have equivalent series resistances (ESRs). All these add damping and affect the ripple attenuation. Here we analyse the 4th-order low-pass filter, in the case of negligible transformer capacitances. Resistances \( r_{dc} \) and \( r_{ac} \) should be considered in series with the dc and ac windings respectively, and \( r_{c1} \) and \( r_{c2} \) in series with \( C_1 \) and \( C_2 \) respectively. The transfer function of the filter is

\[
\frac{V_q}{V_n} = \frac{Z_2 Z_4}{Z_1 Z_2 + Z_2 Z_3 + Z_2 Z_1 + (Z_1 + Z_2) Z_4}
\]

in which

\[
\begin{align*}
Z_1 &= s k \sqrt{L_{ac} / L_{dc}} \\
Z_2 &= s (L_{ac} - k \sqrt{L_{ac} / L_{dc}}) + r_{ac} + r_{c1} + 1 / s C_1 \\
Z_3 &= s (L_{dc} - k \sqrt{L_{ac} / L_{dc}}) + r_{dc} \\
Z_4 &= r_{c2} + 1 / s C_2
\end{align*}
\]

Substituting (12) into (13), applying the null condition (4) and simplifying yields a fourth-order transfer function

\[
\frac{V_q}{V_n} = \frac{1 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4}
\]

in which the coefficients are

\[
\begin{align*}
a_1 &= C_1 (r_{ac} + r_{c1}) + C_2 r_{c2} \\
a_2 &= C_1 C_2 r_{c2} (r_{ac} + r_{c1}) \\
b_1 &= C_1 (r_{ac} + r_{c1}) + C_3 (r_{dc} + r_{c2}) \\
b_2 &= L_{ac} C_1 + L_{dc} C_2 + C_1 (r_{ac} + r_{c1}) (r_{dc} + r_{c2}) \\
b_3 &= [L_{ac} (r_{ac} + r_{c1}) + L_{dc} (r_{dc} + r_{c2})] C_1 C_2 \\
b_4 &= (1 - k^2) L_{ac} L_{dc} C_1 C_2
\end{align*}
\]

The high-frequency asymptotic gain falls by 40dB/decade.

The consequences of parasitic resistances are best evaluated numerically. Their general effect is to damp resonances, worsen high-frequency attenuation, and reduce the depth of notches. In fact if there is too much parasitic resistance, the notches may not appear.

V. SYNTHESIS OF “ZERO-RIPPLE” CONVERTERS

It is very convenient to treat Fig. 1 as a general building block when formulating low-ripple converters. The smoothing transformer can be used at the input or the output, or in
the middle (such as in the dc link of an ac-dc-ac converter). The blocking capacitor can be returned to any suitable point, not necessarily to ground. It can even be taken to the output capacitor [18], if increased output ripple is acceptable.

Boost-converter applications have been widely presented in the literature [10], [15]. In [17], discrete modeling techniques similar to the established methods in [3] are used to discuss coupled filters in a variety of dc-dc converters, including a SEPIC.

The conventional ripple cancellation method for the Cuk converter can be redrawn in terms of Fig. 1 to show the nature of the coupled interaction. Fig. 5 shows four steps that illustrate the process. Fig. 5a shows the base converter, with the smoothing transformer building block in the place of the inductors. In Fig. 5b, the circuit is redrawn to create a pi network in the center. Provided the smoothing capacitors are selected for resonance at or below the switching frequency, the equivalent tee for the pi circuit has an inductive center leg and capacitive upper sections. The resulting pi to tee conversion is shown in Fig. 5c (the coupling lines are left out for clarity). The final step of isolation produces the well-known “zero-ripple” Cuk converter shown in Fig. 5d. Other low-ripple cases can be developed much along the same lines.

A. Measuring $k$

For successful implementation, it is vital to know the coupling coefficient $k$. Here we give two methods for measuring $k$ experimentally. All measurements must be taken at a low enough frequency that parasitic capacitance is negligible. The methods do not need calibrated inductance measurements, as they depend only on ratios, although it is useful to perform the measurements with a dc current source in place to set the correct bias level.

Once the open-circuit winding inductances $L_1$ and $L_2$ are known, together with their coupling coefficient $k$, the coupled inductor is represented in SPICE by lines like

```
L1 1 2 10mH
L2 3 4 20mH
k12 L1 L2 0.98
```

1. Open/short-circuit inductance method

The inductance of winding 1 is measured with winding 2 open-circuited ($L_{1,sc}$) and short-circuited ($L_{1,sc}$). The coupling coefficient is calculated from

$$k = \sqrt{1 - \frac{L_{1,sc}}{L_1}}$$

A second value of $k$ can be obtained by measuring winding 2, and the two averaged. This method is particularly suitable when $k$ is close to unity, as it then gives an accurate result despite possible difficulty in measuring $L_{1,sc}$. However, it is unsuitable when the winding resistances are substantial.

2. Series aiding/opposing inductance method

In this method, four inductance readings are taken. First, with the other winding open-circuited, the individual winding self-inductances are measured ($L_1$ and $L_2$). Then the two windings are connected in series and their combined inductance is measured, in series-aiding connection ($L_{aid}$) and series-opposing ($L_{opp}$). (Note that $L_{aid} > L_{opp}$ always, and that $L_{aid} - L_{opp} = 4M$). The coupling coefficient is calculated from

$$k = \frac{L_{aid} - L_{opp}}{4\sqrt{L_1L_2}}$$

An advantage is low sensitivity to winding resistance. This method is unsuitable when $k$ is small, as in that case it depends on the difference between two similar quantities.

B. Experimental Test Circuit

Fig. 6 shows a buck converter configured for a coupled smoothing filter. As in most practical transformers, the inductor coupling is close to unity. The series aiding/opposing method yields $k = 0.98$, although a lower value would be expected under dc excitation. The inductance ratio $\sqrt{L_{aid}/L_{dc}}$ was found to be 0.998. We would expect an error of a few percent in the matching condition (4) based on these values. On the other hand, since $k$ is less than the ratio,
(9) will support a notch (at the switching frequency for instance). The nominal switching frequency was 50 kHz.

The circuit was tested in the laboratory and was compared to PSPICE simulations. Figs. 7-10 give some key results. Fig. 7 shows experimental results with no coupling. In this case, the inductor is the only filter, and the expected triangular ripple appears. The upper trace is the output voltage at 2 V/div, while the lower trace is inductor current at 0.5 A/div.

Fig. 8 shows results with coupling in place, with a capacitor of 10 µF or more (little change was observed over the tested range from 10 µF to 4700 µF). The upper trace, still the output voltage at 2 V/div, shows large reduction in the ripple. More significant, the ripple is now a square wave rather than a triangle, which suggests a series resistance effect. Eq. (9) indicates that 10 µF is sufficient to establish the coupled filter as a low-pass combination, consistent with the measured result. The lower trace is the ac winding current $i_{L2}$ at 0.5 A/div. As expected, this winding carries the triangular ripple (plus an ESR jump).

Based on (9), the capacitance needed for notch filtering at the switching frequency is 4.2 µF. To test this, a 4.4 µF ceramic capacitor was used, then the switching frequency was tuned for the best match. The waveform is shown in Fig. 9. The result at 49 kHz was a much reduced ripple. The upper trace is the ac-coupled output voltage at 0.2 V/div. There is some residual switching frequency, but it appears as a sequence of pulses with little energy in between. The RMS ripple is only about 25 mV. Current $i_{L2}$ remains triangular (at 0.5 A/div) to cancel the ripple.

C. Simulation Comparisons

Fig. 10 shows a SPICE simulation of the test circuit with a 4.2 µF capacitor in the coupled filter. The match is very good, largely because the various resistive parasitics were included in the simulation.
VII. CONCLUSION
Coupled magnetic filters can be applied to low-ripple dc-dc converter design. However, limited understanding of coupling methods has led to repeated reinvention of basic coupled-filter building blocks. Designers of high-performance converters can benefit from a systematic analysis of coupled filtering alternatives.

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